PHYC 581: High Energy Astrophysics
Fall 2018

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Thick Accretion Disks:

Thin disks emit the released gravitational energy efficiently via radiation, and this is the reason why they are geometrically thin.

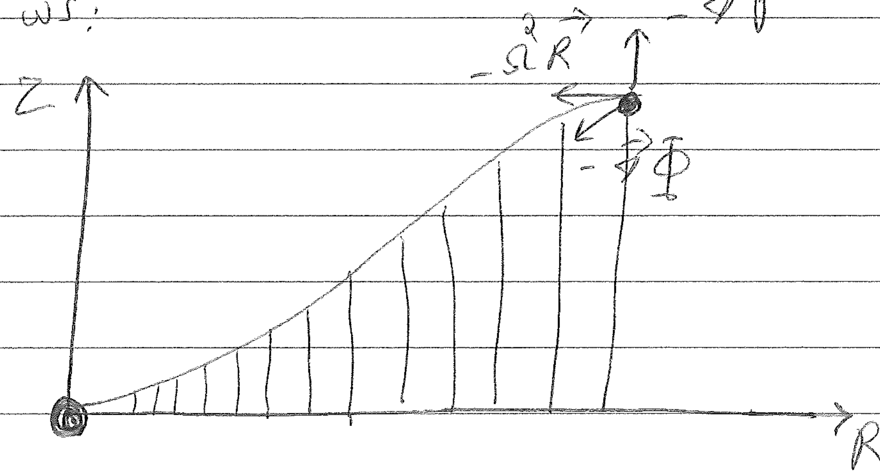
There is another accretion disk scenario that is likely to apply to AGN's and QSO's. In AGN's, the accretion disk may be a geometrically thick one for at least two reasons. It may radiate inefficiently, in which case the orbiting plasma retains much of the dissipated energy. Also, an AGN disk may be accreting at a super-Eddington rate, in which case the transfer of radiation through the optically thick medium cannot keep up with the heating rate. In both cases, the disk is prevented from collapsing to the equatorial plane, and hence thick.

Let us start with the structure of thick disks. In these systems the vertical structure becomes important. It is easy to see that pressure gradient plays an important role in dynamics. For example, consider an axi-symmetric configuaration as follows:

$v_R \simeq 0$

$v_\phi \simeq R\Omega$

$v_z \simeq 0$



Then:

$$\frac{1}{\rho}\vec{\nabla} P = -\vec{\nabla}\Phi + \Omega^2 \vec{R}$$

It is seen from the figure that one cannot get rotation about the z axis without the $\vec{\nabla} P$ term. This implies that the rotation will not be keplerian, i.e., $\Omega = \Omega(R,z) \neq \Omega_K(R)$.

The surface of the disk, represented by the relation $z = z_s(R)$, is an isobaric surface, $\vec{\nabla} P|_s = 0$, which results in:

$$-\vec{\nabla}\Phi\big|_s + \Omega^2\vec{R}\big|_s = 0 \Rightarrow (\Omega^2 R)_s = \left(\frac{\partial \Phi}{\partial z}\right)_s \frac{\partial z_s}{\partial R_s} + \left(\frac{\partial \Phi}{\partial R}\right)_s$$

To illustrate some important differences arising for a thick disk, let us consider a simple situation where the disk's boundary is a cone:

$z_s(R) = \pm(\tan\alpha) R$

Then:

$$\Phi = \frac{-GM}{(R^2+z^2)^{1/2}} \Rightarrow \frac{\partial \Phi}{\partial z} = \frac{GMz}{(R^2+z^2)^{3/2}} \Rightarrow \left(\frac{\partial \Phi}{\partial z}\right)_s = \frac{GM\tan\alpha\cos^3\alpha}{R^2}$$

$$\frac{\partial \Phi}{\partial R} = \frac{GMR}{(R^2+z^2)^{3/2}} \Rightarrow \left(\frac{\partial \Phi}{\partial R}\right)_s = \frac{GM\cos^3\alpha}{R^2}$$

Putting things together, we find:

$$(\Omega^2 R)_s = \frac{GM\tan^2\alpha\cos^3\alpha}{R^2} + \frac{GM\cos^3\alpha}{R^2} = \frac{GM\cos\alpha}{R^2}$$

Thus:

$$\Omega^2(R) = \frac{GM\cos\alpha}{R^3}$$

Therefore, in this particular case, the disk is Keplerian, but with a reduced mass $M\cos\alpha$. The reduction is due to the opposing effect of pressure gradient from the gas. We note that $\Omega \to \Omega_K$

as $d \to 0$, which is expected in the thin disk limit.

We now discuss how the disk shape can affect the luminosity.

Recall that the radiative flux is given by:

$$\vec{F}_{rad} = \frac{-c}{\kappa \rho} \vec{\nabla} P \qquad (\kappa: \text{opacity})$$

In the case of a disk, it can be written as:

$$\vec{F}_{rad} = \frac{c}{\kappa} \vec{\nabla} \Phi - \frac{c}{\kappa} \Omega^2 \vec{R}$$

The luminosity is given by: (after using Gauss's law)

$$L = \frac{c}{\kappa} \oint \vec{\nabla} \Phi \cdot d\vec{s} - \frac{c}{\kappa} \oint \Omega^2 \vec{R} \cdot d\vec{s} = \frac{c}{\kappa} \int_V \nabla^2 \Phi \, dV - \frac{c}{\kappa} \int_V \vec{\nabla} \cdot (\Omega^2 \vec{R}) \, dV$$

Poisson's equation for the gravitational field states that:

$$\nabla^2 \Phi = 4\pi G \rho$$

Then:

$$L = \frac{c}{\kappa} \int_V 4\pi G \rho \, dV - \frac{c}{\kappa} \int_V \vec{\nabla} \cdot (\Omega^2 \vec{R}) \, dV = \frac{4\pi c G M}{\kappa} - \frac{c}{\kappa} \int_V \vec{\nabla} \cdot (\Omega^2 \vec{R}) \, dV$$

The first term on the right-hand side is the Eddington luminosity for mass M. To see the effect of the second term, we write:

$$\vec{\nabla}\cdot(\Omega^2\vec{R}) = \frac{1}{R}\frac{d}{dR}(\Omega^2 R^2) = 2R\Omega\frac{d\Omega}{dR} + 2\Omega^2$$

Since $\Omega^2 = \frac{GM\cos\alpha}{R^3}$ for our simplified geometry, we have:

$$\vec{\nabla}\cdot(\Omega^2\vec{R}) = -\Omega^2$$

Hence:

$$L = L_{edd} + \frac{c\,GM\cos\alpha}{\kappa}\int_V \frac{1}{R^3}\,dV \qquad (dV = 4\pi R^2\tan\alpha\; dR)$$

Finally, we arrive at the following expression:

$$L = L_{edd}\left[1 + \sin\alpha\,\ln\left(\frac{R_2}{R_1}\right)\right]$$

Here R_1 and R_2 are the inner and outer radii of the funnel region in the thick disk respectively. As an example, let us tak $\alpha = 45°$ and $R_2 = 100 R_1$. We then have $L \sim 4 L_{edd}$. It is therefore seen that an AGN can emit at over 4 times the Eddington limit. We emphasize that the vertical extension in these structures comes as a direct result of the gravitational dissipation associated with such large accretion rates, as mentioned before.

As a side note, we learned at the very beginning that if the accretion rate exceeds the Eddington limit, the accretion will be heavily suppressed due to outward radiation pressure. However, this is strictly true for a spherically symmetric geometry. For a thick disk configuration, the Eddington limit may be circumvented as we just saw.